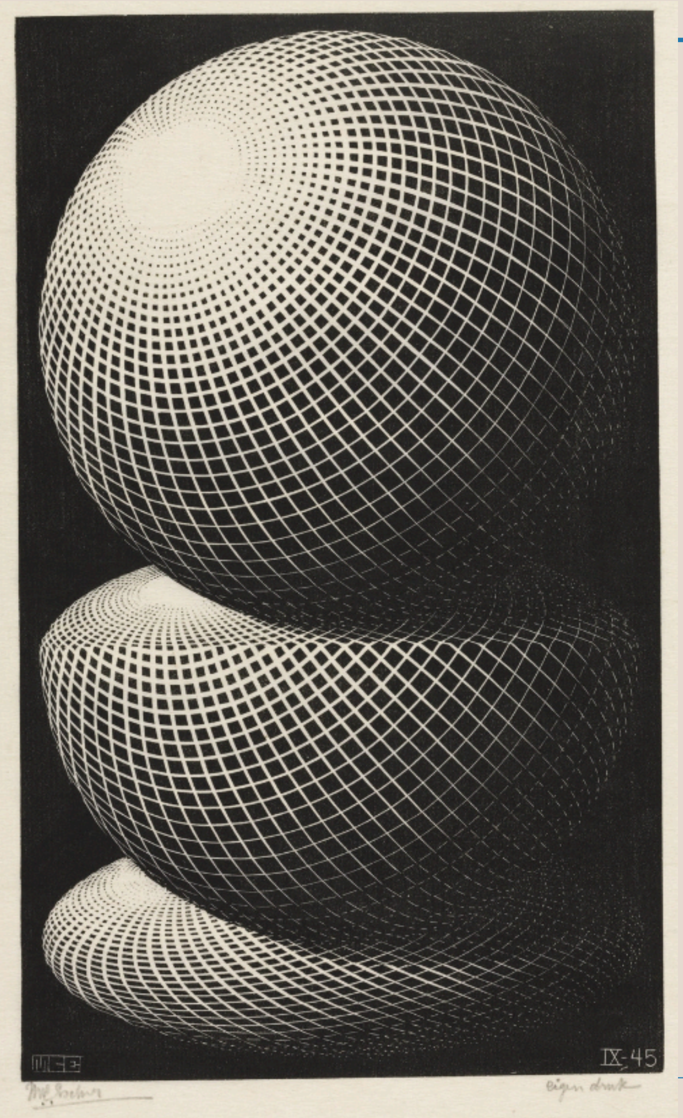
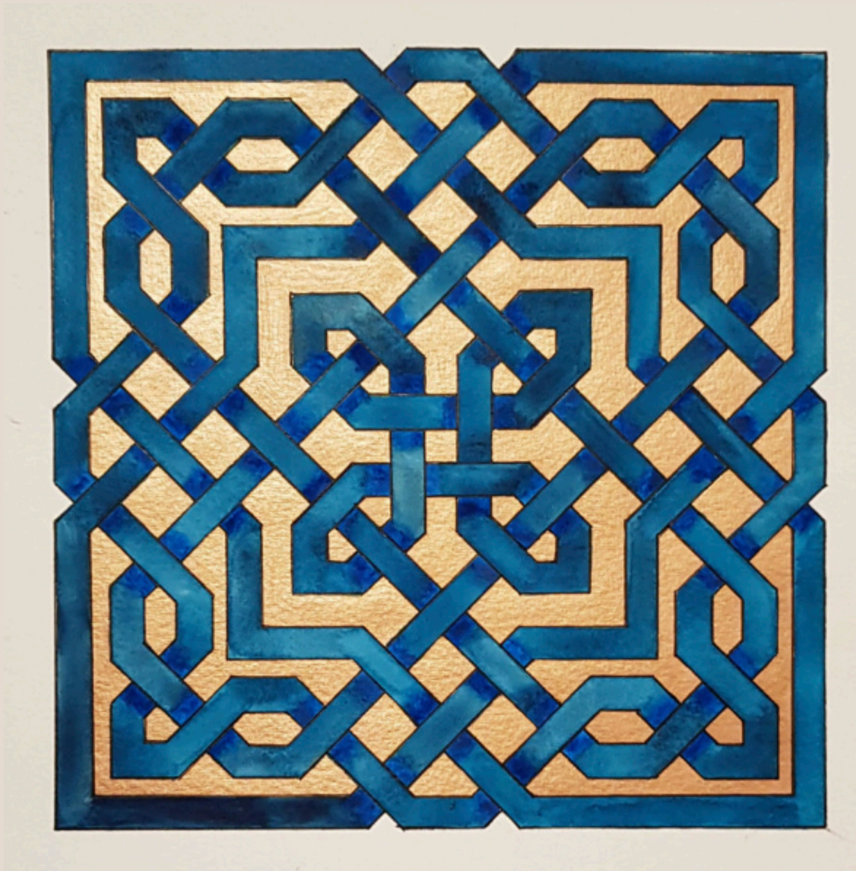
Quantum Computer Science
Spring 2024

Saeed Mehraban





Logistics:

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Gradescope code: ZW8E3Y
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Grades:
10% Class Participation
20% Midterm Exam
30% Final Exam
40% Problem Set

What is quantum computing?

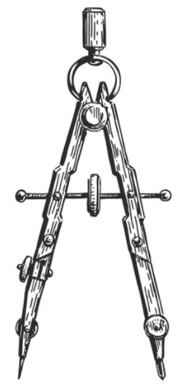
"It is a new paradigm of computing
based on physical devices
that harness quantum mechanical laws."

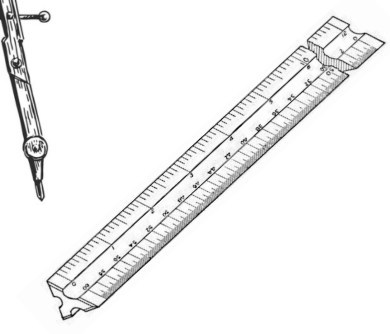
Our plan forward:

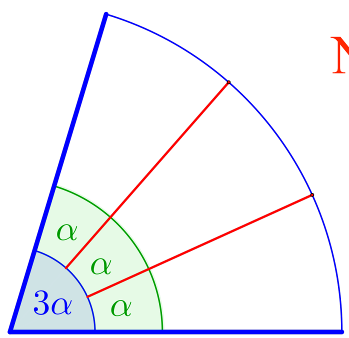
- Digital Computations
- Quantum Mechanics
- Quantum Computations
- Quantum Algorithms
- Quantum Error Correction

What is computation:

<u>Ancient example</u>: Constructible lengths



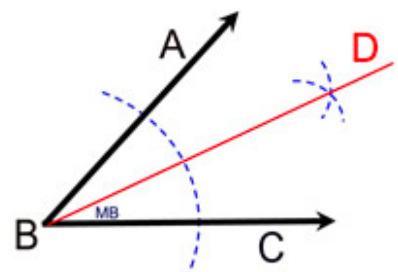




Non constructible

$2^{1/3}$

heptagon



bisector

Constructible

$\sqrt{2}$

15-gon

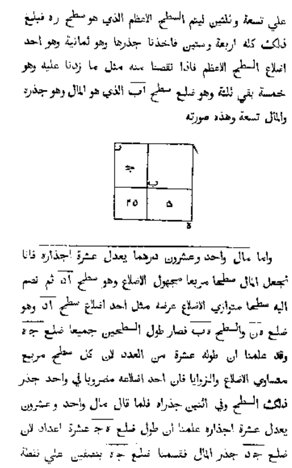
<u>2nd Example</u>:

Algorithm as a algebraic description of computation

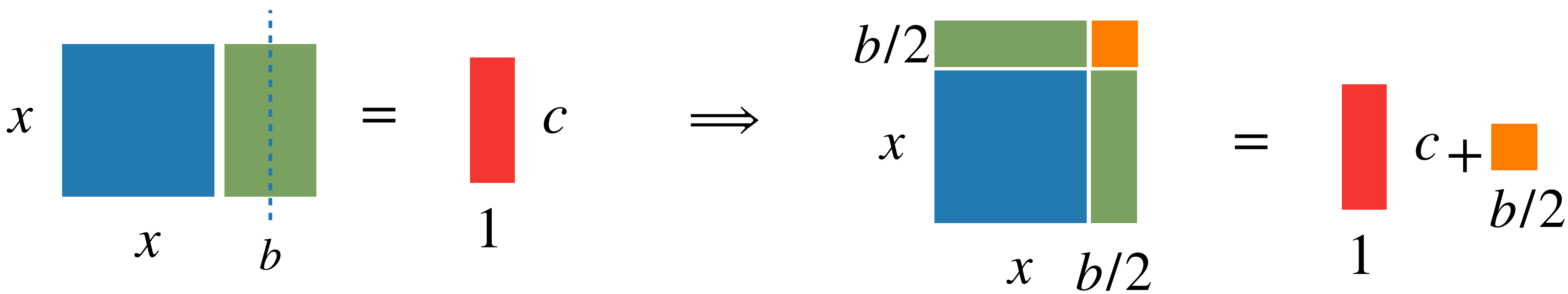
$$x^2 + bx = c$$

$$\Longrightarrow \; (x + b/2)^2 = (b/2)^2 + c$$

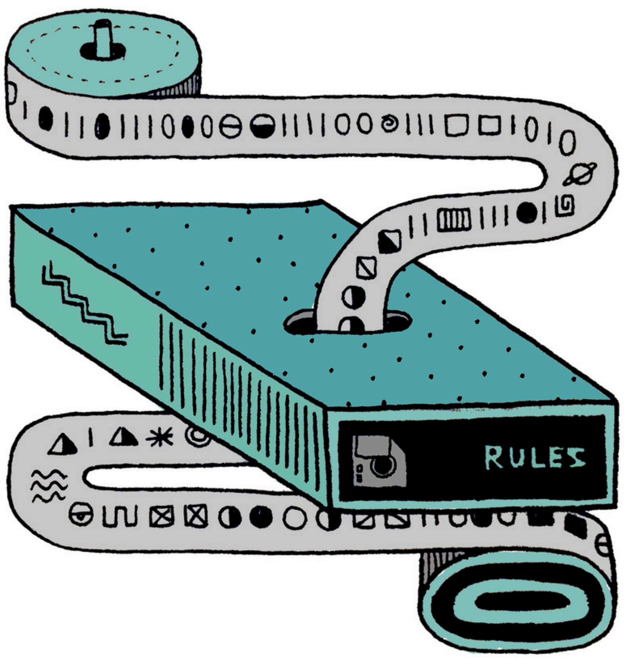
Kharazmi (c. 800)

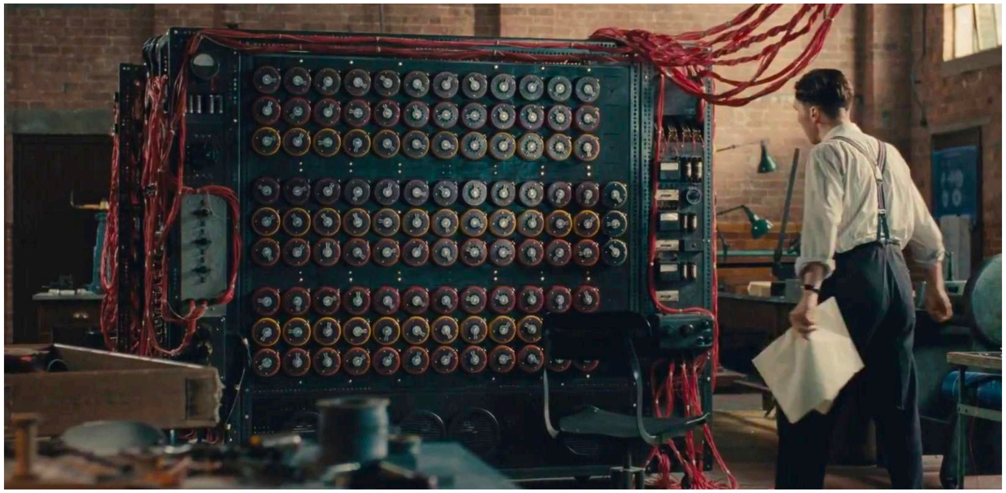

على تسعة وثلثين ليتم السطح الاعظم الذي هو سطح ره فبلغ
ذلك كله اربعة وستين فاخذنا جذرها وهو ثمانية وهو احد
اضلاع السطح الاعظم فاذا نقصنا منه مثل ما زدنا عليه وهو
خمسة بقي ثلثة وهو ضلع سطح اب الذي هو المال وهو جذره
والمال تسعة وهذه صورته

واما مال واحد وعشرون درهما يعدل عشرة اجذاره فانا
نجعل المال سطحا مربعا مجهول الاضلاع وهو سطح اد ثم نضم
اليه سطحا متوازي الاضلاع عرضه مثل احد اضلاع سطح اد وهو
ضلع هن والسطح هب فصار طول السطحين جميعا ضلع جه
وقد علمنا ان طوله عشرة من العدد لان كل سطح مربع
متساوي الاضلاع والزوايا فان احد اضلاعه مضروبا في واحد جذر
ذلك السطح وفي اثنين جذراه فلما قال مال واحد وعشرون
يعدل عشرة اجذاره علمنا ان طول ضلع هج عشرة اعداد لان
ضلع جد جذر المال فقسمنا ضلع جه بنصفين على نقطة

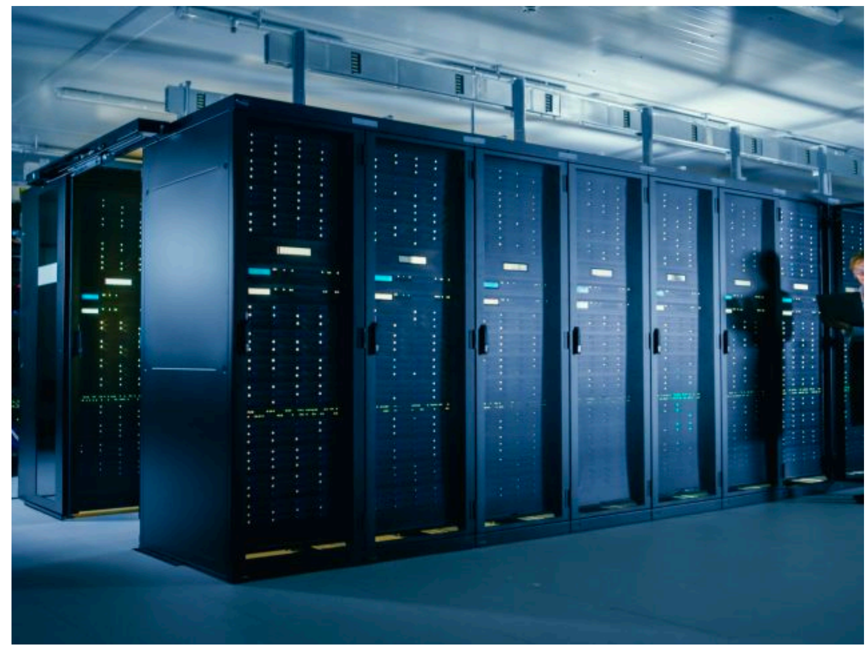


3rd Example **Turing machines**



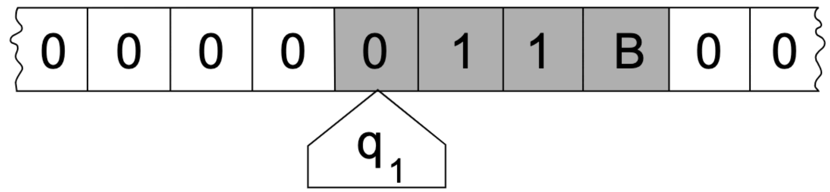


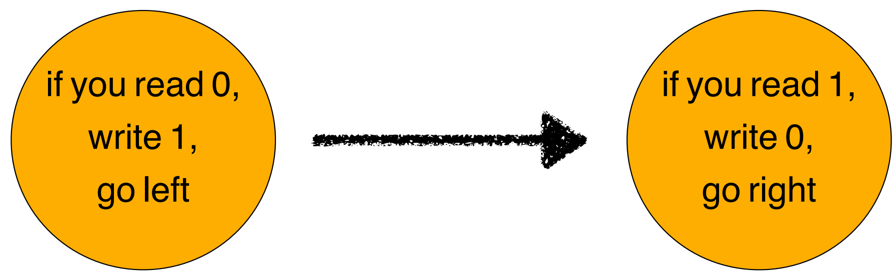
Modern example



Ed Fredkin:

Turing machine is like a mathematician who is writing down a mathematical proof on a paper





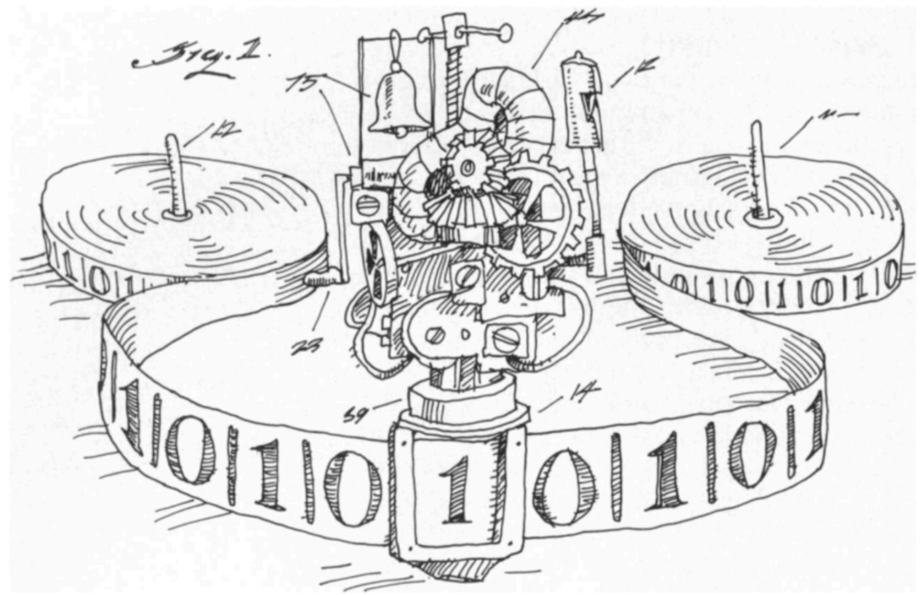
Almost (!) equivalent to digital circuits

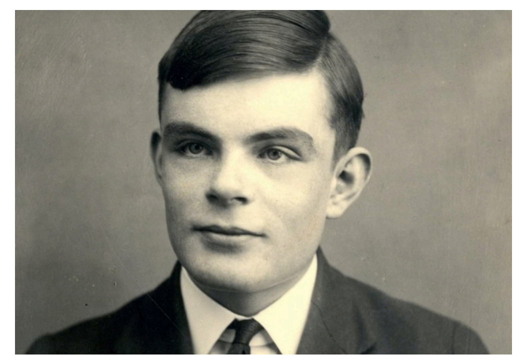
The Church-Turing Thesis:

All means of performing computations are equivalent to Turing machines.

Note: Turing machines follow classical physics!

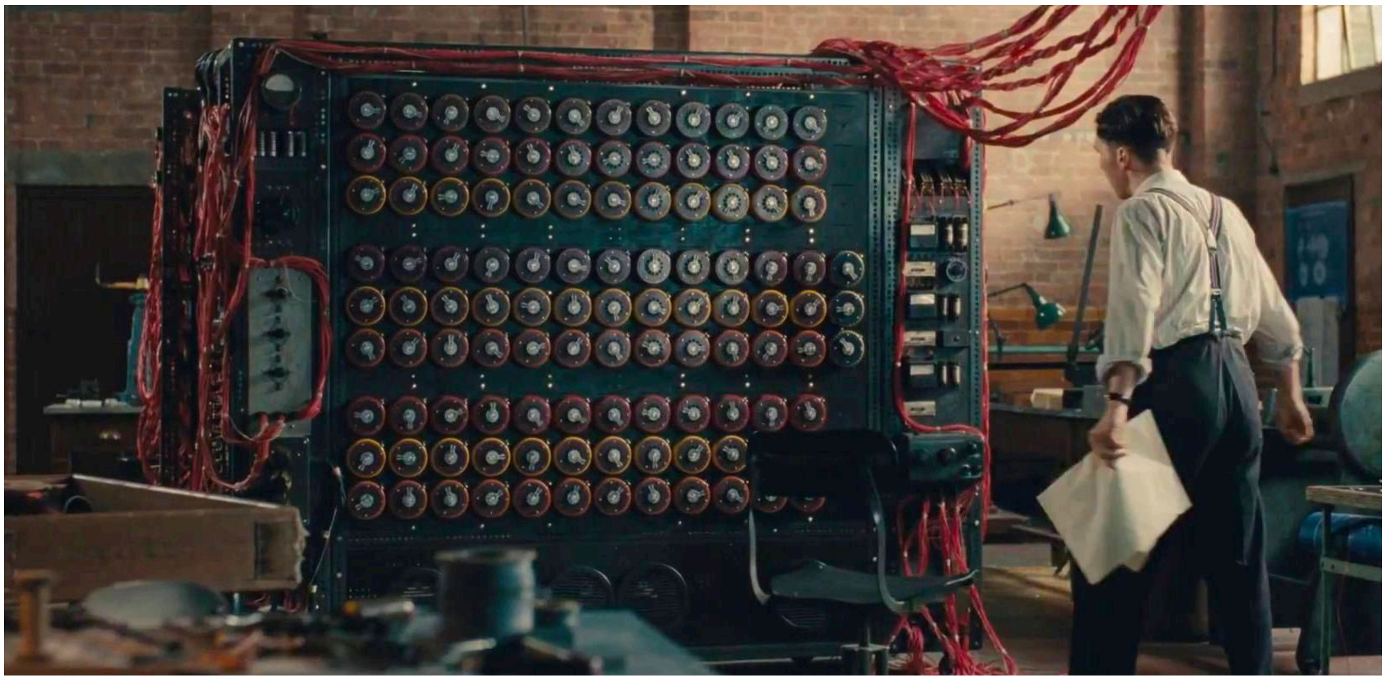
Are the laws governing the physical world equivalent to the classical mechanics?





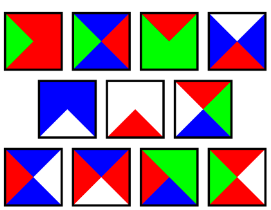
- Halting problem: Given a Turing machine, decide if it ever halts!

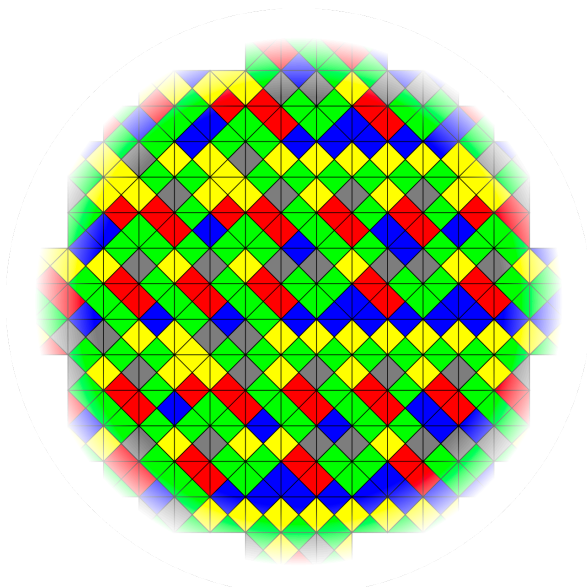
“Halting problem is undecidable!”



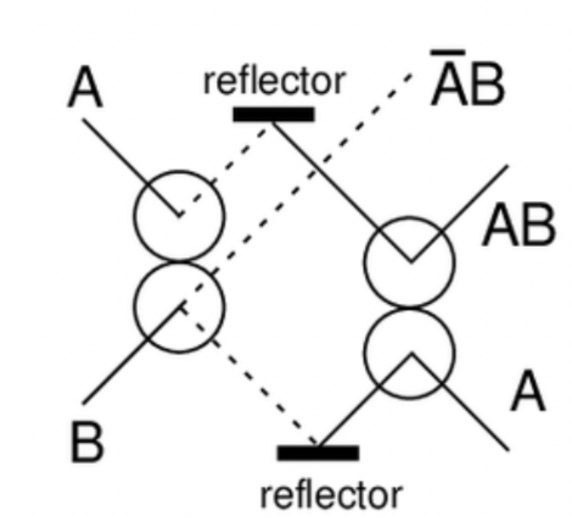
- Computation as a physical process

Wang tiles: You can encode arbitrary computations using these colored tiles





Billiard balls have equal computing power to the Turing machine!



- Big-O notation. Exponential growth vs. Polynomial growth

We say $f(n) = O(g(n))$, if there exist n_0, c such that for any $n \geq n_0, f(n) \leq c \cdot g(n)$.

Polynomial growth:

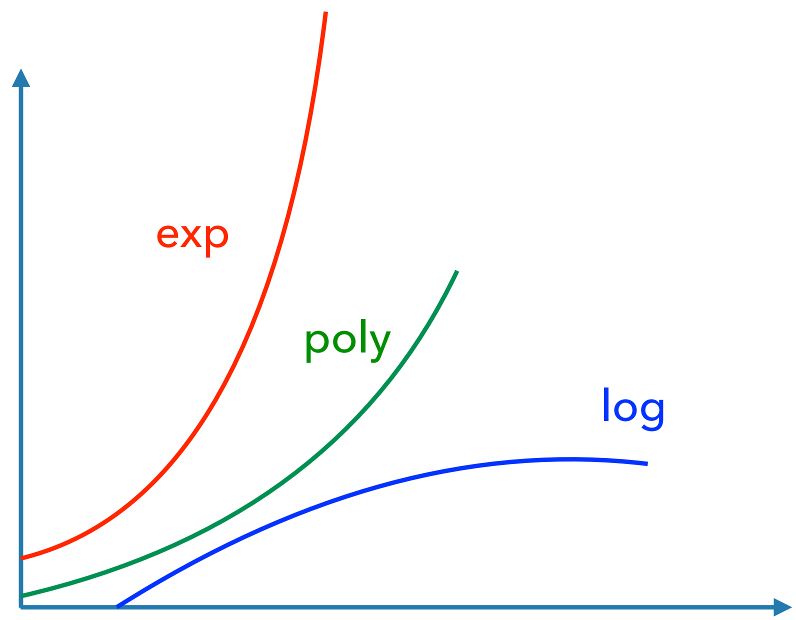
$f(n) = O(n^d)$, constant d.

Exponential growth:

$g(n) = 2^{O(n^d)}$, constant d.

Logarithmic growth:

$h(n) = O(\log^d n)$, constant d.



- Extended Church-Turing Thesis

"All computational machines are efficiently equivalent."

Efficient mean polynomial time equivalence.

Example: Factoring composite numbers.

Problem: Given an n digit composite number, find one of its factors.

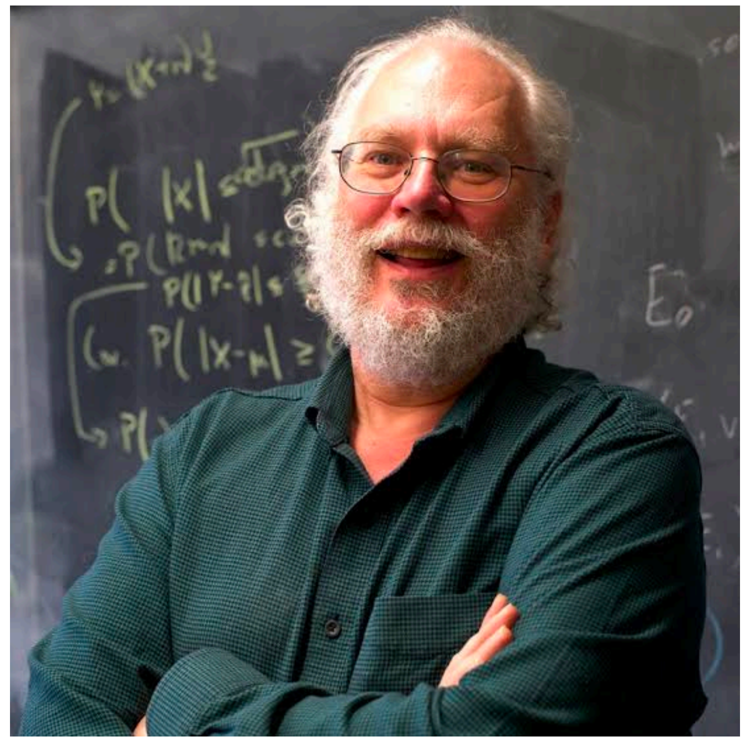
$$499242563 = 971 \times 514153$$

The best algorithm known for this problem (based on number field sieve) runs in time $2^{O(n^{1/3})}$. For a 3000 digit number, it takes the age of the universe to solve this problem.

A way of challenging the extended Church-Turing thesis is by giving a polynomial time algorithm for this problem.

Quantum computers as a way of challenging the extended Church-Turing thesis

In 1994 Peter Shor gave a polynomial-time quantum algorithm for the Factoring problem



Quantum computers: Computational devices which harness quantum mechanical laws.

Quantum Mechanics:

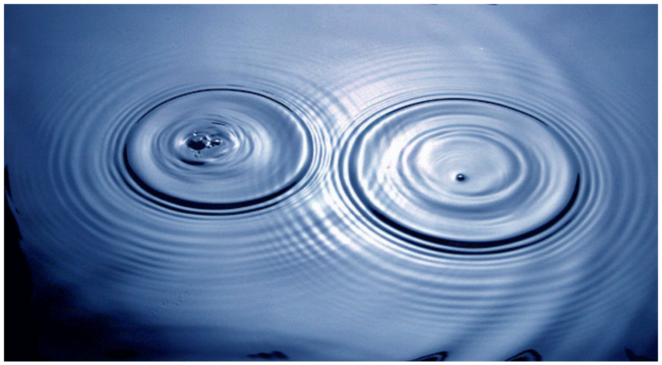
1. Subatomic particles
2. Wave-particle duality
3. Interference phenomenon
4. Entanglement
5. Energy is Quantized

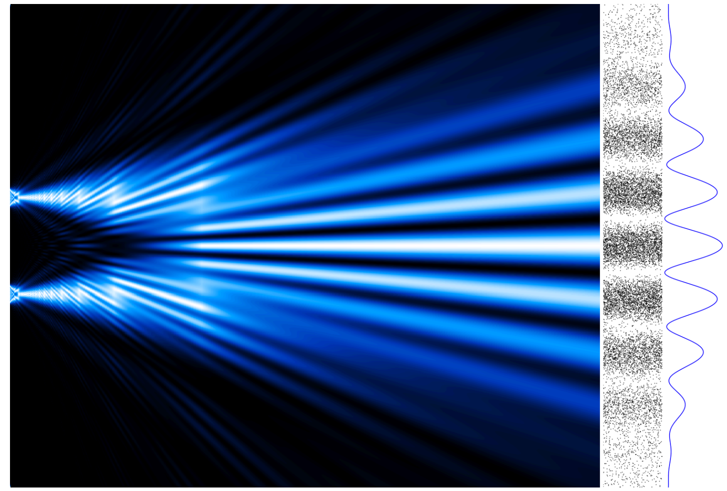
Particle

Wave

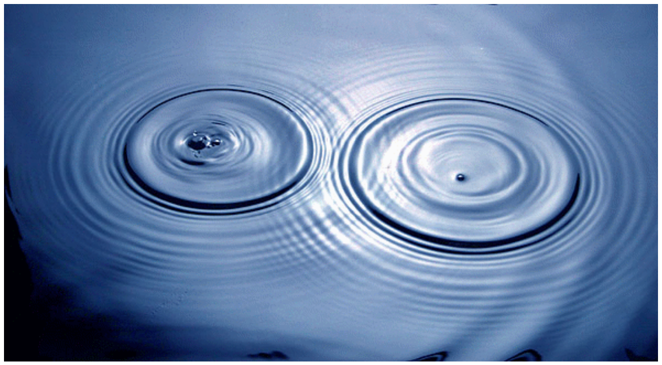
credit: Colm Gorey

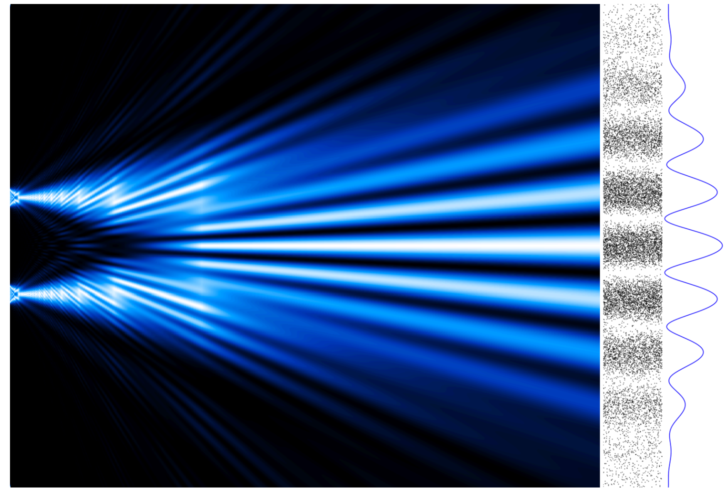
Wave-particle duality

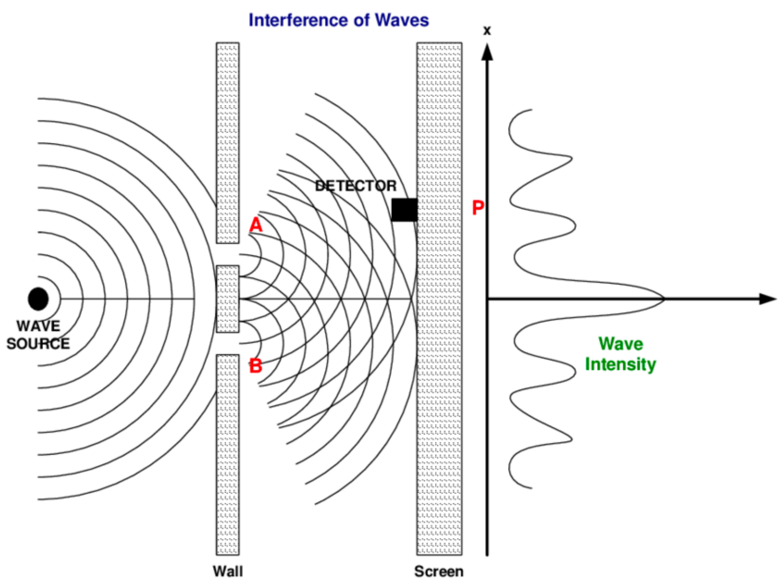




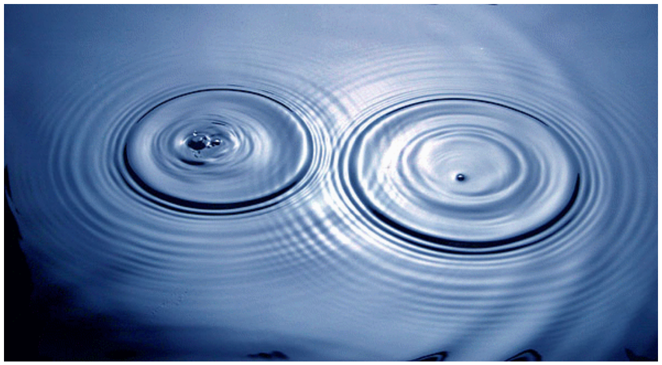
Wave-particle duality

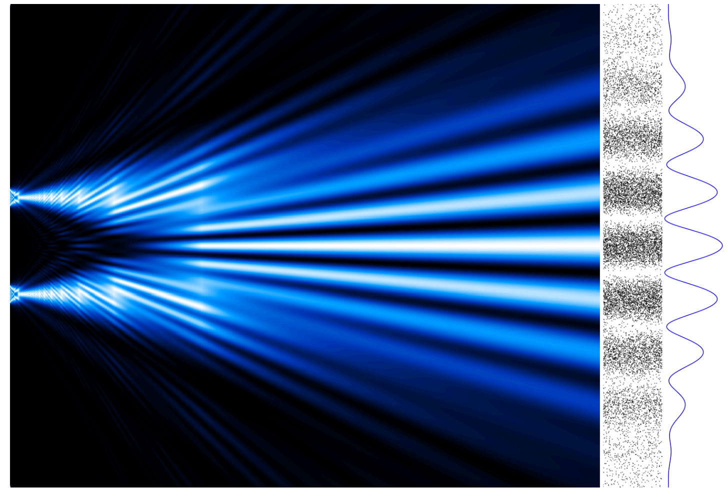


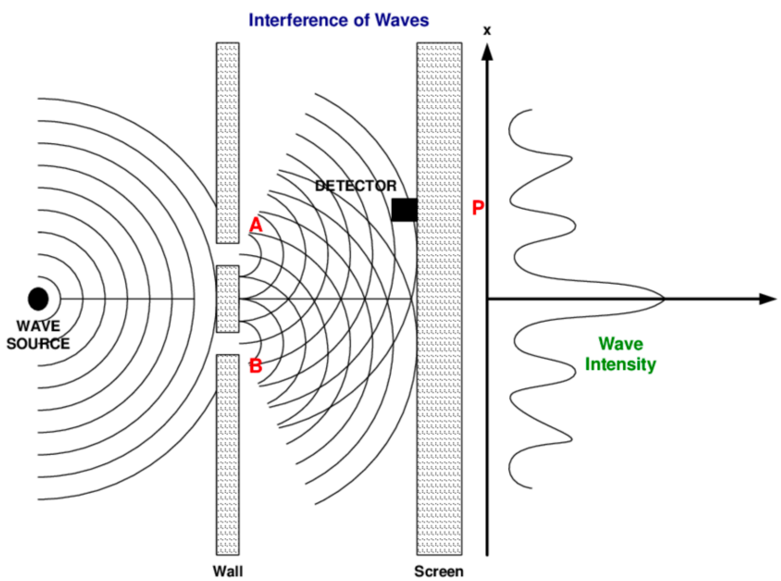


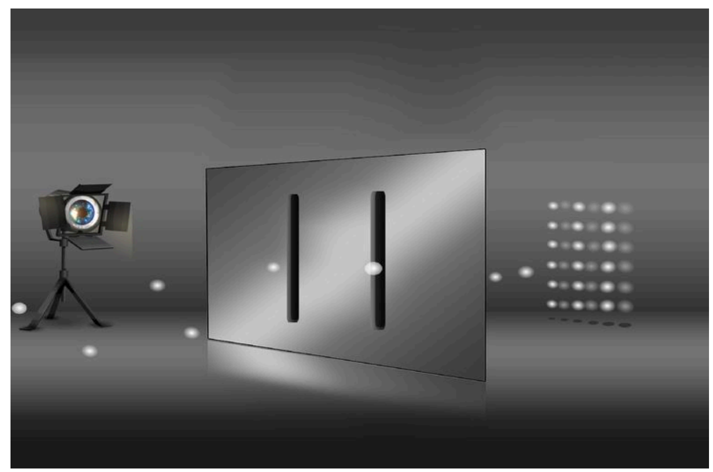


Wave-particle duality







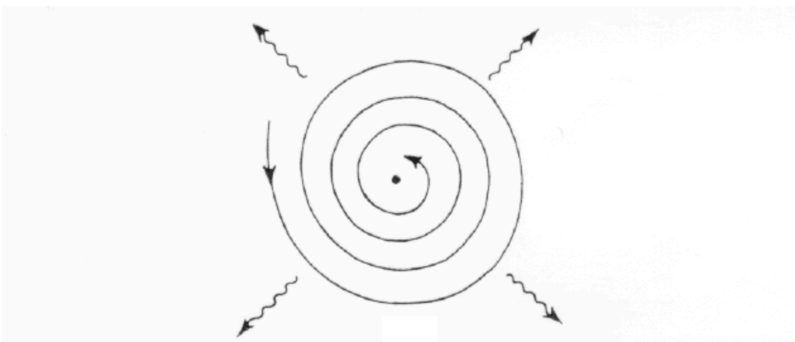


Wave-particle duality

Stability of materials

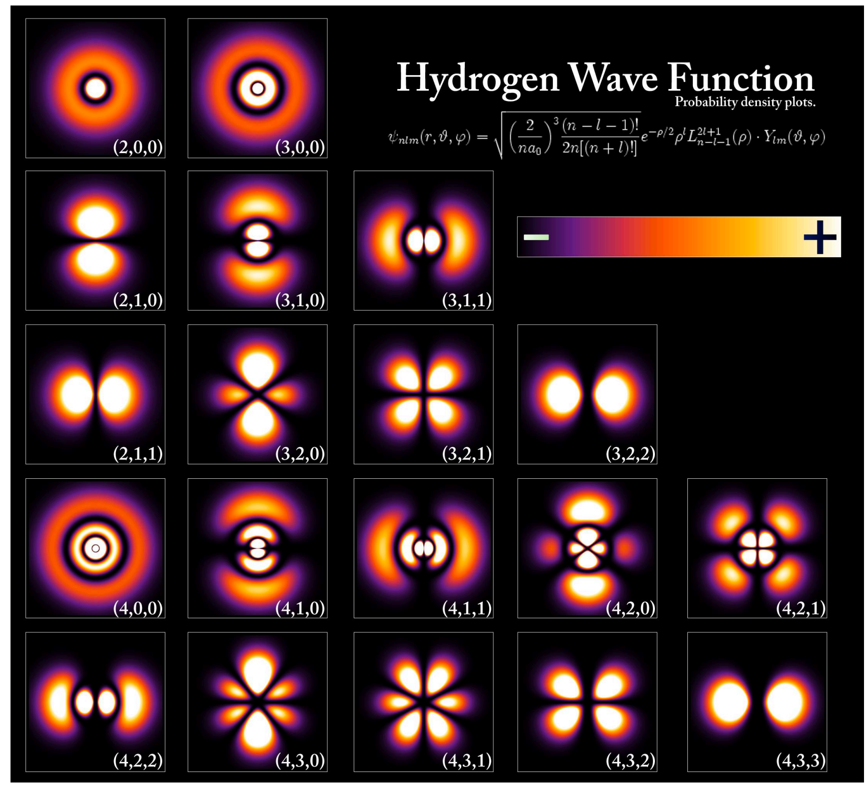
Quantum mechanics predicts stable and quantized solutions

Classical mechanics predicts atoms should collapse within 10^{-12}seconds

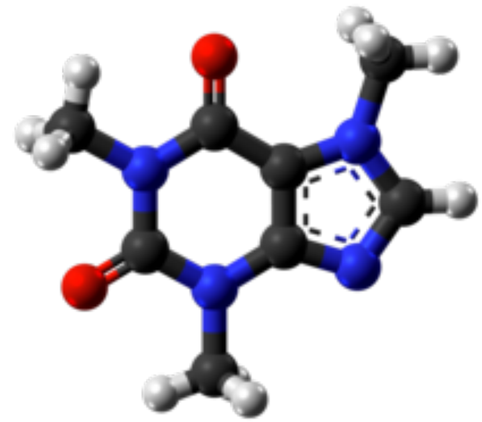


According to classical physics, an electron in orbit around an atomic nucleus should emit electronmagnetic radiation (photons) continuously, because it is continually accelerating in a curved path. The resulting loss of energy implies that the electron should spiral into the nucleus in a very short time (i.e. atoms can not exist).

Credit: uoregon.org



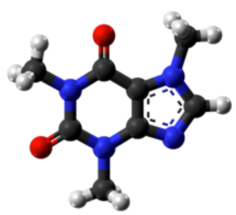
It was clear since the early days of quantum mechanics that simulating many-body quantum system takes exponentially-long computations



Richard Feynman: If simulating quantum systems is so difficult, let's build a computer out of quantum mechanical elements!

<u>**Quantum Algorithms**</u>

Fast simulation of molecules



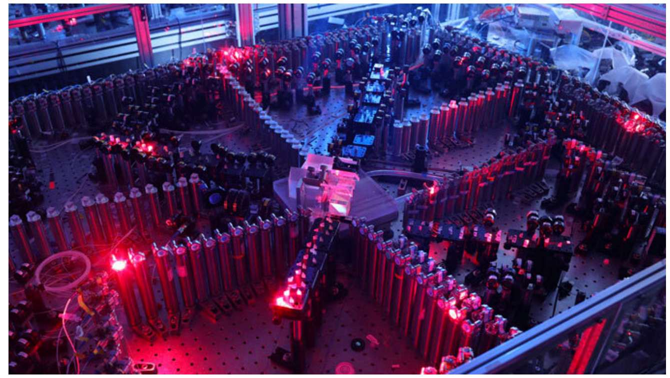
Designing drugs or special materials

Fast factoring

8674238671342341 = ?????????7 x ????????????

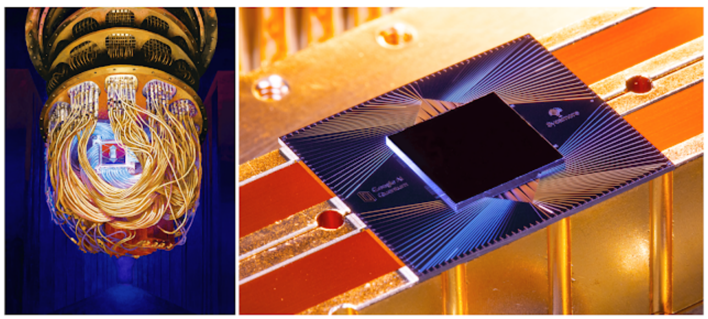
Breaking the RSA code

Fast search!



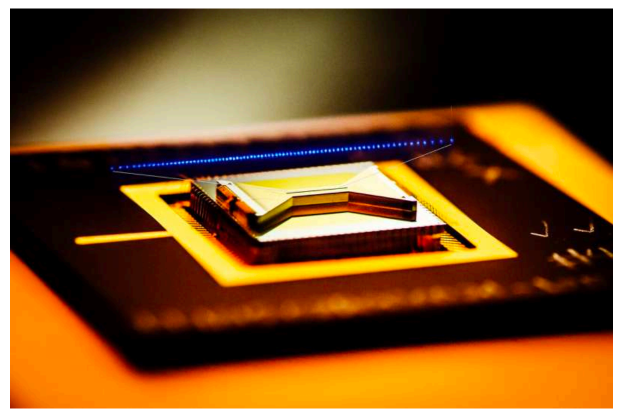
Based on light

credit: ustc.edu



Based on superconducting Qubits

credit: Google AI

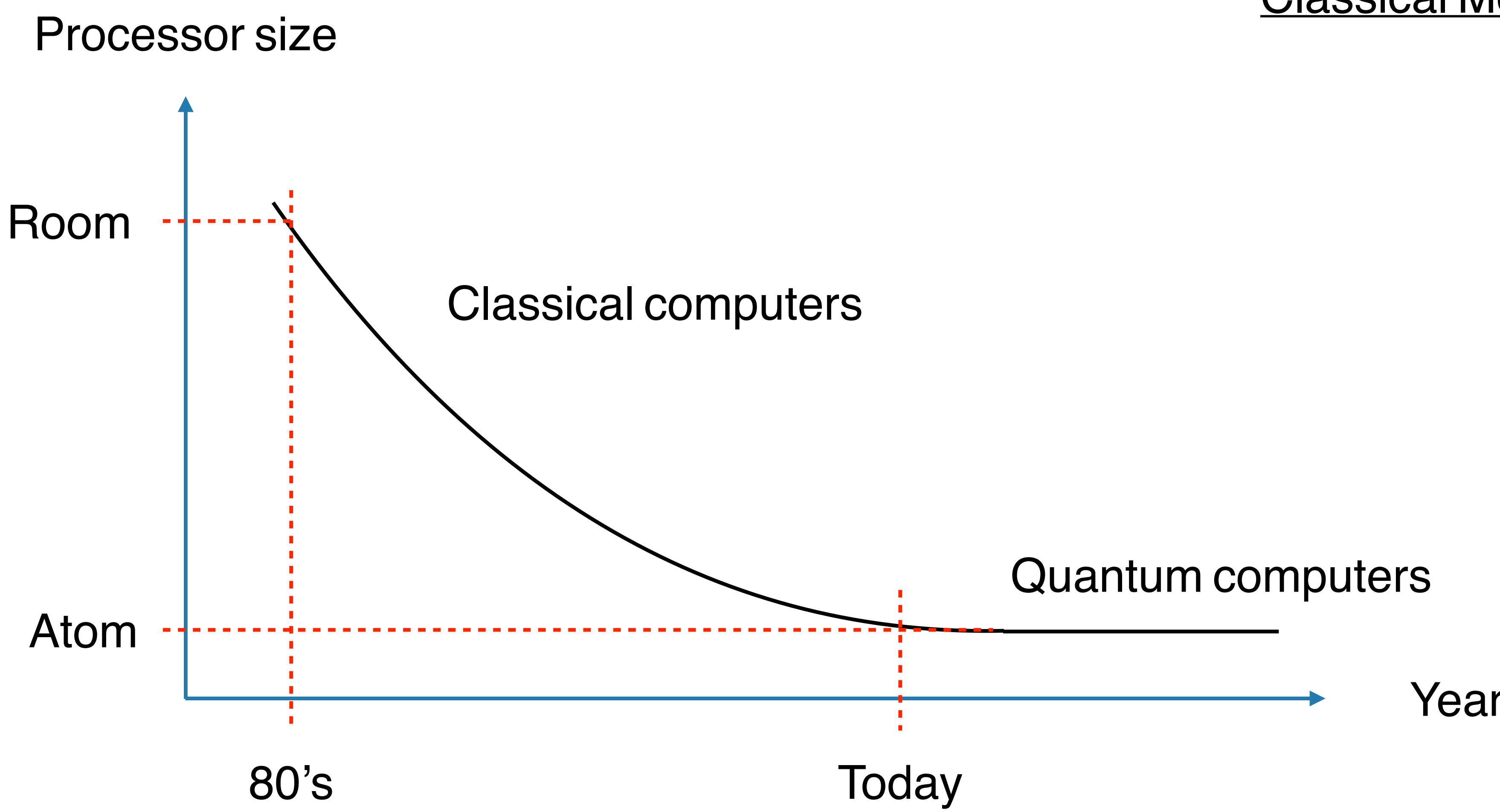


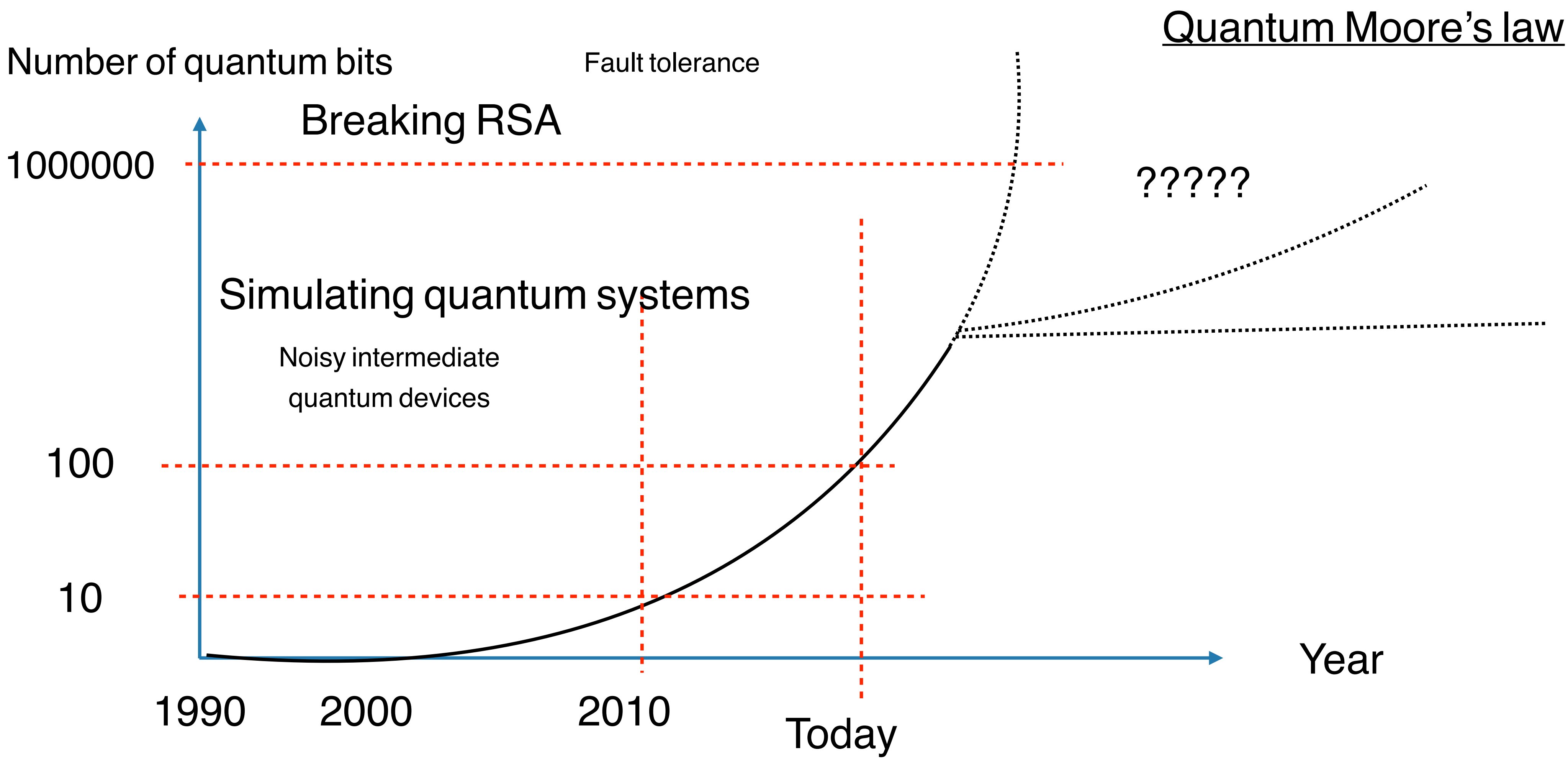
Based on trapped ions

Duke University

Recent implementations

Classical Moore's law





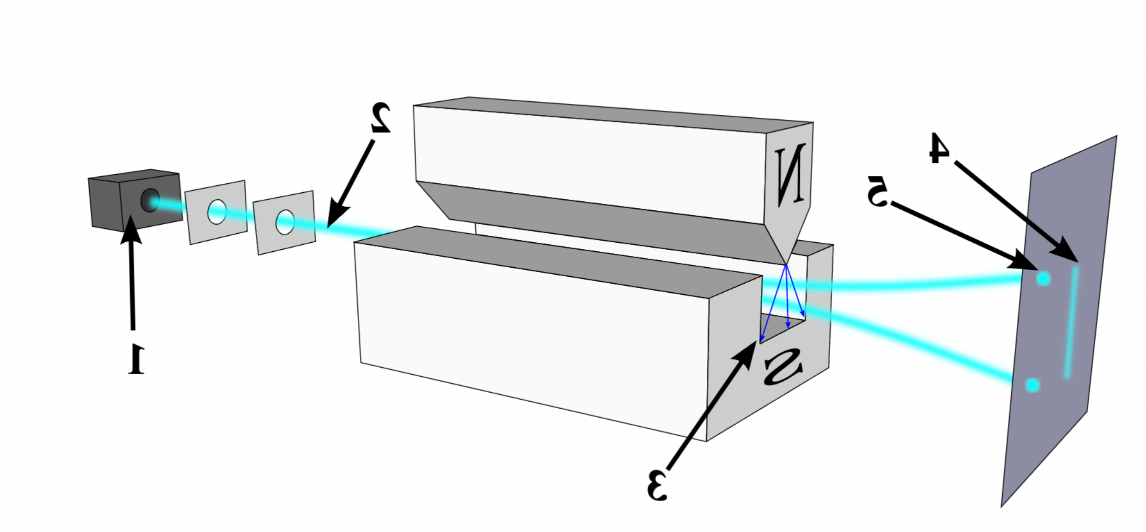
Quantum Moore's law
Number of quantum bits
Fault tolerance
Breaking RSA
1000000
?????
Simulating quantum systems
Noisy intermediate
quantum devices
100
10
Year
1990
2000
2010
Today

Why is building a quantum computer so difficult?

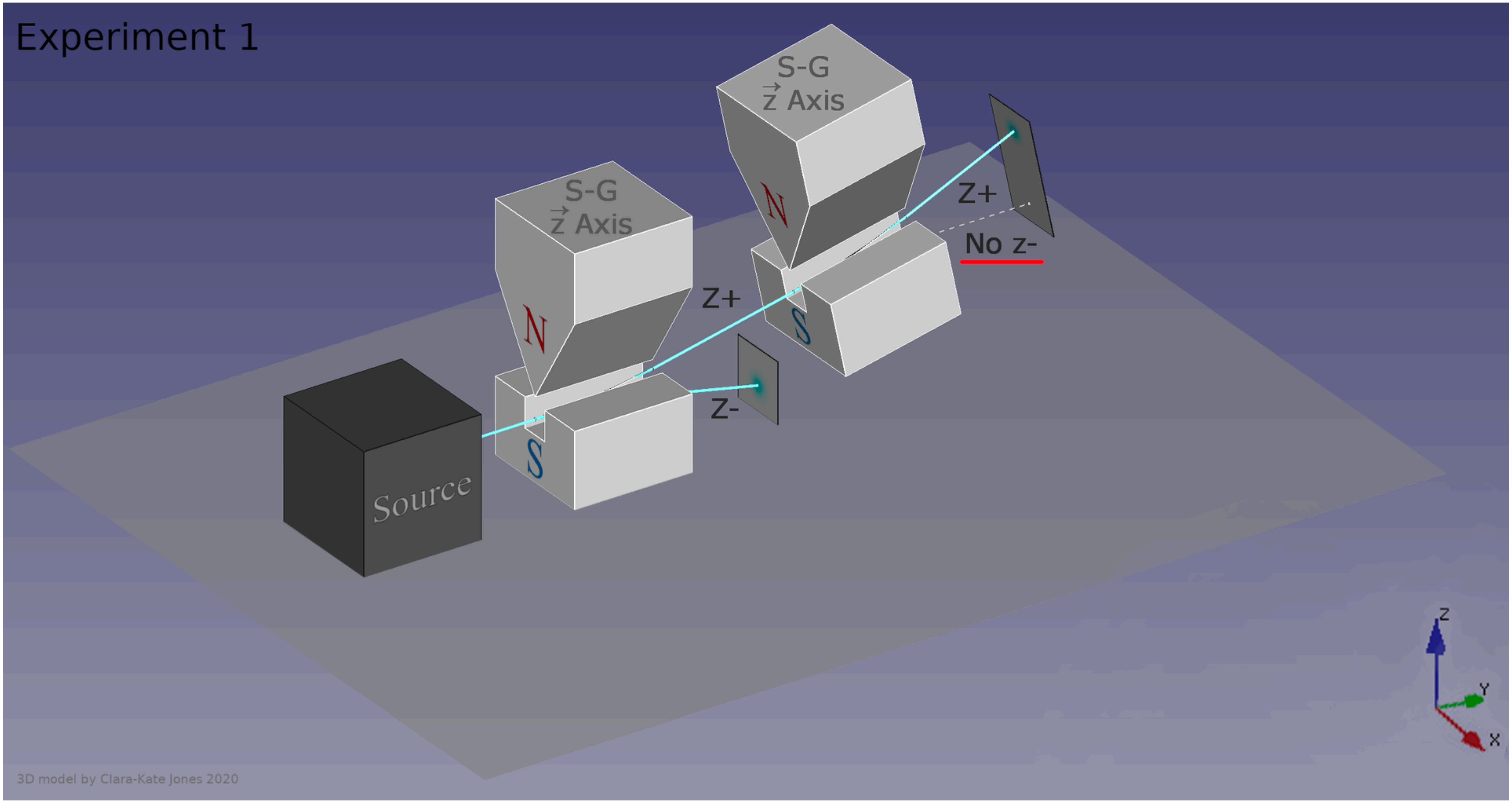
We are writing information at atomic scales. There are no pens in there!

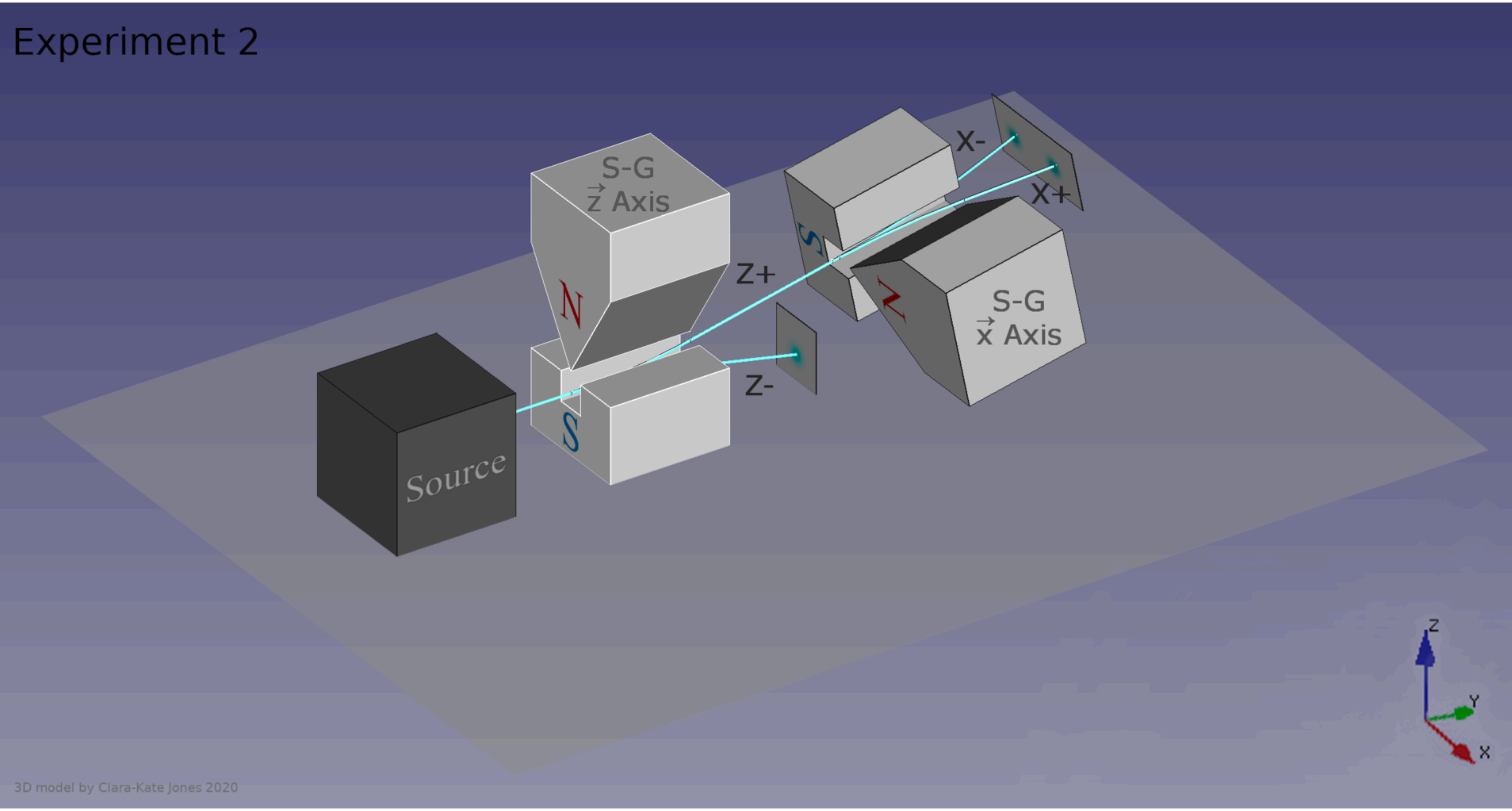
Solution: Fault-tolerance and error correction

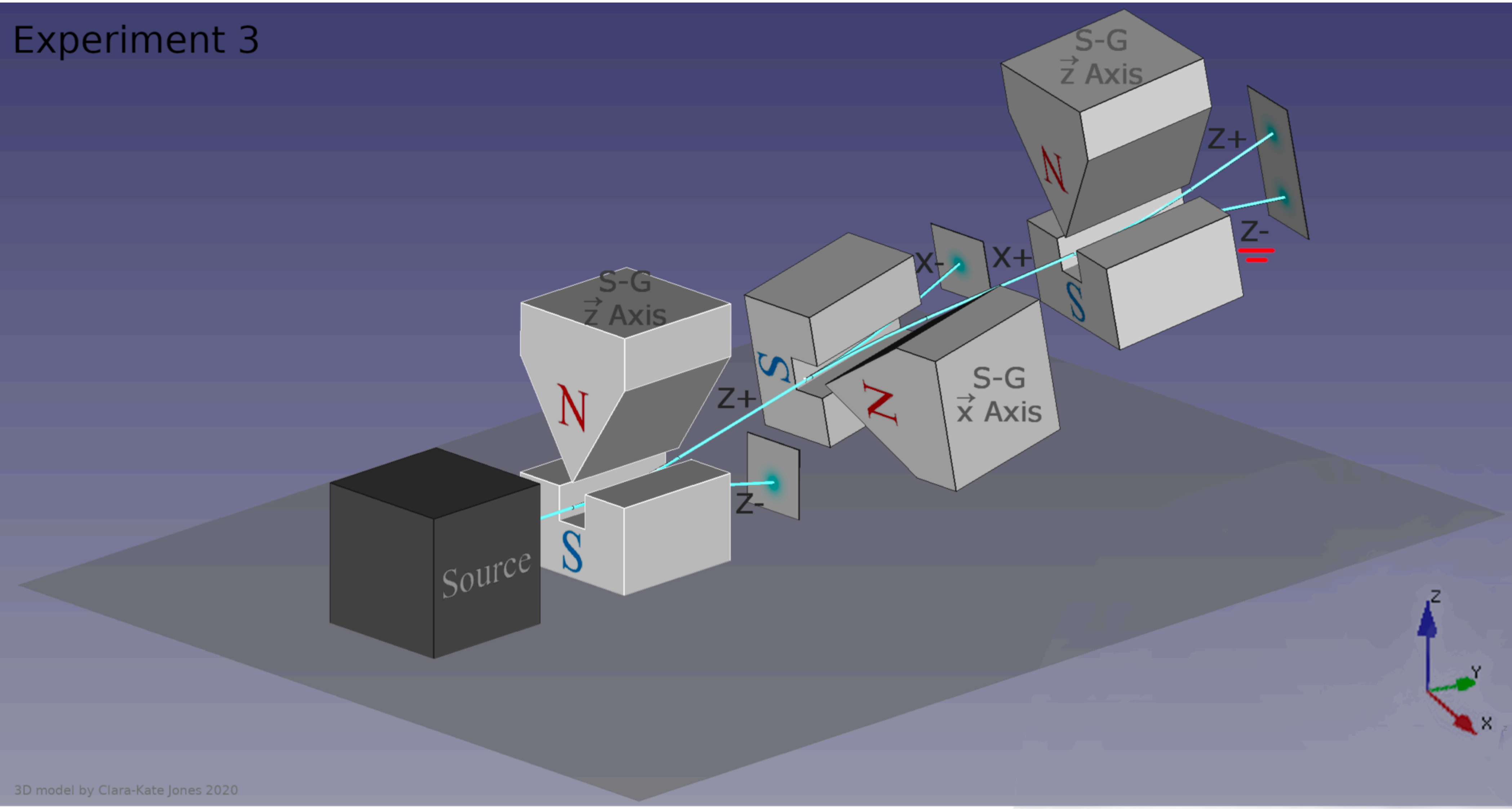
Stern-Gerlach experiment



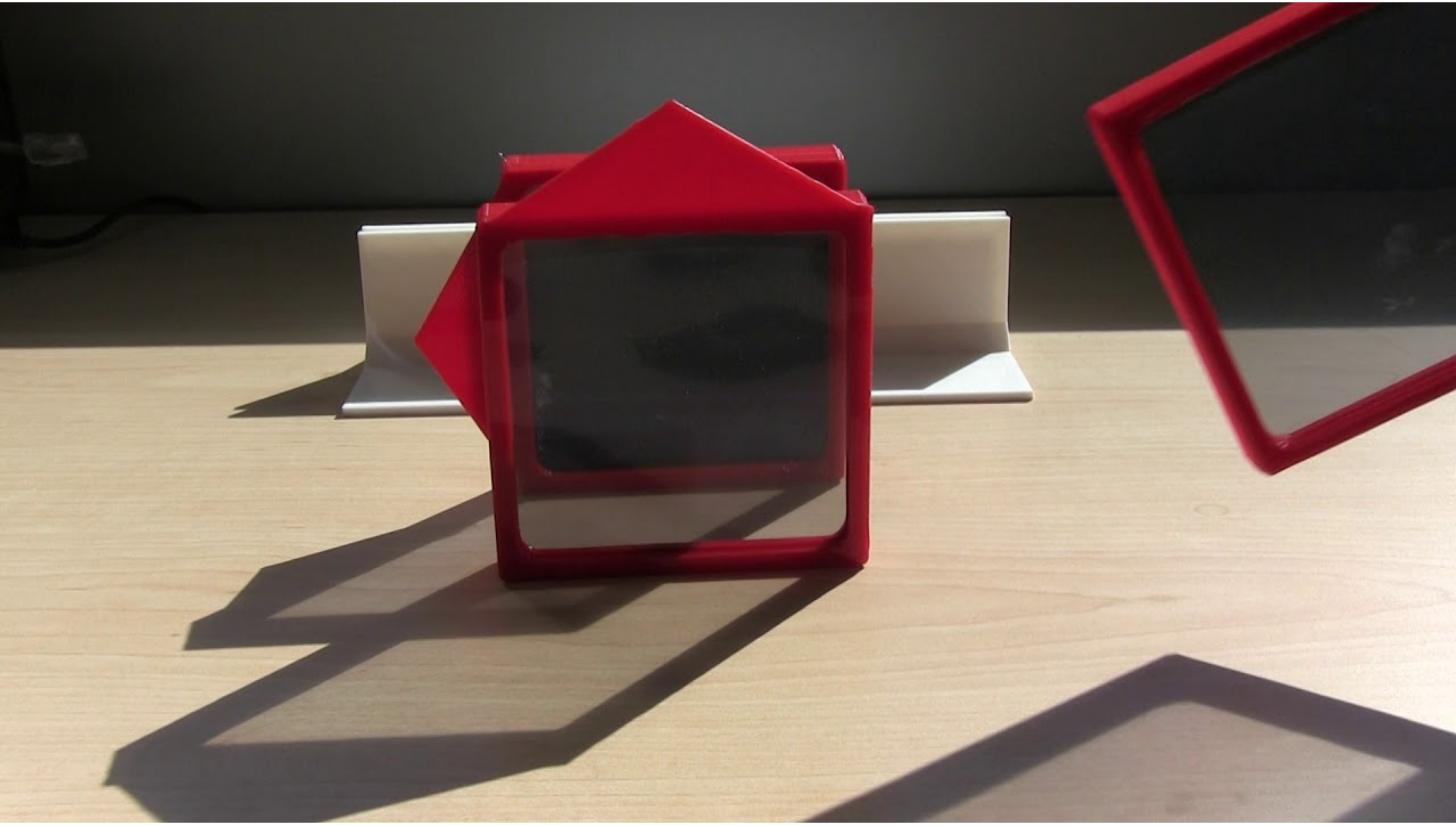
Watch this video https://en.wikipedia.org/wiki/Stern-Gerlach_experiment


Experiment 1
S-G
$\vec{z}$ Axis
N
S
Source
Z+
Z-
S-G
$\vec{z}$ Axis
N
S
Z+
No z-
3D model by Clara-Kate Jones 2020
Z
Y
X


Experiment 2
S-G
$\vec{z}$ Axis
N
S
Source
Z+
Z-
X-
X+
S
N
S-G
$\vec{x}$ Axis
Z
Y
X
3D model by Clara-Kate Jones 2020



Experiment 3
S-G
z Axis
N
S
Source
Z+
Z-
S
N
S-G
x Axis
X-
X+
S-G
z Axis
N
S
Z+
Z-
Z
Y
X
3D model by Clara-Kate Jones 2020



Third polarizing filter experiment

Link: https://www.youtube.com/watch?v=5SIxEiL8ujA